Description of Meniscus Profiles in Free Coating

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The authors are interested in describing flow fields in dynamic menisci. To predict the fields, the complete location of the curved interfacial boundary must be known or predicted. Thus the following question arises: What is the size and shape of the entire meniscus? Several authors have reported important flow effects for a portion of film profiles, for such cases as bubble flow in a tube (Cox, 1962), meniscus rise in a tube (Ludviksson and Lightfoot, 1968), or drainage of drops (Denson, 1970). There is, however, a need for describing the complete profiles.

The geometrical case considered here is the liquid film formed on a flat sheet by continuous, vertical withdrawal from a large bath of wetting liquid. Experiments by Van Rossum (1958) and others have shown that the meniscus thickness h on the sheet decreases asymptotically from the liquid level of bath to a constant thickness h_0. Taking x as the distance above the bath surface, the problem becomes one of describing how the flow thickness $h(x)$ is influenced by coating speed U_w, fluid properties (μ, ρ, σ), and bath depth. One important independent group is the nondimensional speed called the capillary number $Ca \equiv U_w(\mu/\sigma)$.

The qualitative photographs of Soroka (1969) show that flow deformations of meniscus thickness can be quite large. The pioneering work of Groenveld and Van Dortmund (1970) correlated data for the upper part of the meniscus with an expression having two parameters, a size quantity, and a slope. They also predicted the slope of upper meniscus for high Ca but did not predict the size of the meniscus. In this paper, the complete meniscus profile is described for a viscous liquid and the influences of speed and bath depth are noted.

EXPERIMENT

Data were obtained using a continuous belt apparatus described previously (Soroka and Tallmadge, 1971) for entrainment studies. As before, the liquid bath was located between the two pulleys and a wiper was used on the backside of the belt. Tests showed that flow around the side of the belt was negligible at the conditions studied here. Quantitative profile data were obtained with a motor oil (Fluid B) having the following properties at 26.7°C: viscosity of 1.31 N-s/m² (13.1 poise), surface tension of 0.0327 N/m, and a density of 885 kg/m³. Thus the capillary length "a" is 2.75 mm for this fluid; here $a \equiv (2\sigma/\rho g)^{1/2}$ and "a" equals the maximum static rise.

The meniscus profiles were determined photographically using a 35 mm Nikon camera at a distance of about 200 mm. The profile data were obtained by projecting the negative image on a screen, tracing the expanded profile on paper, and measuring h and x using the scale included in the photograph. The values of the film thickness h_0, which were measured independently using a micrometer as in previous work, confirmed the asymptotic values of h obtained photographically.

The geometry of the bath container is important in the study of bath effects. The horizontal cross section of the container was 325 by 200 mm. Since the 62 mm wide belt was centered, the distance from the belt edge to the side wall was about 70 mm on each side. The back of the belt was about 125 mm from the rear bath wall and the perpendicular distance from the belt to the wall on the free coating side was 200 mm. The oil was returned to the bath about 25 mm from the wall on the coating side. This No. 6 bath container was made with a single side wall, which was uninterrupted for photographic purposes and was removable for assembly purposes. This No. 6 bath was similar in size to that used by Soroka (1969) but differed in construction and assembly. During coating runs, the air temperature was 26.7 ± 0.1°C. Viscosity changes, due to small 2 to 3° changes resulting from viscous heating, were adjusted using temperatures measured in the return liquid.

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DEEP BATH DATA

Profiles were observed at six speeds of 14 to 660 mm/sec in multiples of about two. Thus Ca was varied from about 0.4 to 30. These profiles indicated the large size of the deformations which occurred. At Ca of 12, for example, h = 15 mm at x = 10 mm. These profiles were characterized as deep bath profiles because the bath depth was about 40 times as large as the thickness h_0. For example, at Ca of 0.4, 3, and 12 in runs 312B, 306B, and 302B, the depths used were 40, 92, and 200 mm and the film thicknesses observed were 0.83, 2.39, and 5.02 mm respectively.

The profiles were substantially larger than those predicted by a recent low Ca theory (Lee and Tallmadge, 1972a), probably due to a one-dimensional flow assumption in the theory. A study of two-dimensional flow is in progress (Lee and Tallmadge, 1972b).

SHALLOW BATH DATA

Profiles were also obtained at smaller bath depths. At the 330 mm/sec speed, for example, bath depths of 200, 100, 50, and 25 mm were used.

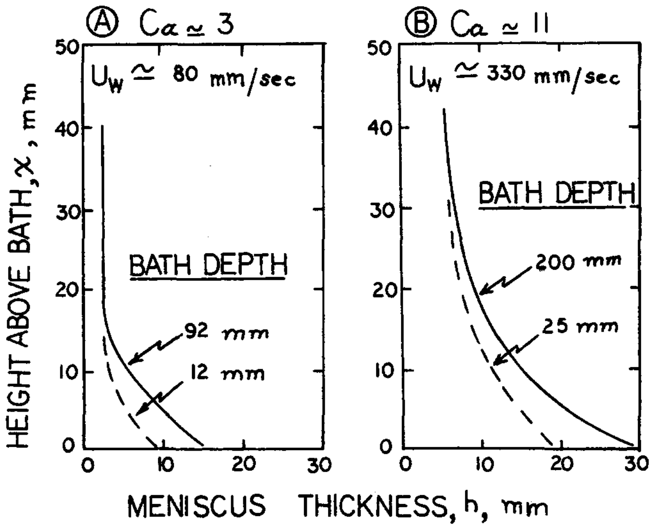

Fig. 1. Effect of bath depth and coating speed on meniscus profiles: Part A at a lower coating speed (runs 306B, 321B), part B at a higher coating speed (runs 302B, 316B), and bath width of 175 mm. Oil viscosity of 1.31 N-s/m²

Sample profiles, shown in Figure 1, show the appreciable reduction in meniscus size which occurs with shallow baths, particularly near the horizontal liquid surface. The reduction, which was appreciable at all speeds studied, indicates the significant role of bath geometry on the size and shape of the meniscus.

UPPER MENISCUS

Groenveld (1970) has reported a stagnation point for a deep bath at a thickness of about $3h_0$. Therefore, the film can be divided about the stagnation point into a thick, lower meniscus region and a thin, upper meniscus region.

Although no analytical expressions have been written for the lower meniscus, some work has been reported for the upper meniscus. Using experimental results from disk coating, Groenveld and Van Dortmund (1970) noted that a data plot of x vs $\ln(h - h_0)$ was linear for the upper-meniscus, thin-film region. Thus the functional relationship for $h(x)$ in the upper meniscus can be written in terms of a slope m and intercept b by

$$x = b - m \ln (h - h_0) \qquad (1)$$

Using $\lambda \equiv x/h_0$ and $L \equiv h/h_0$, the problem becomes one of describing the meniscus profile $L(\lambda)$. In this notation, the profile decreases from a large L at the bath surface ($\lambda = 0$) to $L = 1$ at a large height λ. Equation (1) can be written in nondimensional form as follows, using $M = m/h_0$ and $B = (b/h_0) - M\ln h_0$:

$$\lambda = B - M \ln (L - 1) \qquad (2)$$

The profile data of this work were plotted on the semilog coordinates of Equation (2), as shown in Figure 2. The results indicated that the data can be described by Equation (2) in the upper region but not, in general, in the lower region. A deviation of 45% occurs at $\lambda = 0$ in Figure 2.

Before expressing the data analytically, we note that the b parameter (or B) has little physical meaning here. A more meaningful parameter is that which occurs by extrapolation to the bath surface ($\lambda = 0$), that is, the extrapolated value of thickness, L_B. A more convenient form of Equation (2) is

$$L = 1 + D \exp(-\lambda/M) \qquad (3)$$

Since the extrapolated thickness occurs at $\lambda = 0$, Equation (3) indicates that $L_B = 1 + D$. Thus Equations (1) to (3) can be rewritten as

$$L = 1 + (L_B - 1) \exp(-\lambda/M) \qquad (4)$$

or

$$\frac{h}{h_0} = 1 + \left(\frac{h_B}{h_0} - 1\right) \exp(-x/m) \qquad (5)$$

The upper-meniscus data in Figure 2 are described by the slope $M = 2.08$ and the intercept $L_B = 20.5$. The bath intercept L_B is a useful size parameter because of its direct physical meaning. However, it has not been studied previously. Data taken here indicate that the L_B intercept is a strong function of both Ca and bath depth.

Groenveld and Van Dortmund (1970) have predicted a slope $M = 1.75$ for the upper meniscus in a deep bath, for the case of a high Ca approaching infinity. Their study of disk data indicates that the slope of 1.75 applies at Ca from 10^0 to 10^1 and their overall study suggests that the upper-meniscus slope of 1.75 applies for Ca above one for deep baths.

Deep bath results obtained here for Ca above 1 have slopes which are in general agreement with the 1.75 value. Furthermore, bath depth does not appear to influence slope appreciably. However, preliminary work at lower Ca indicates that slopes are considerably larger at lower speeds and lower fluid viscosities.

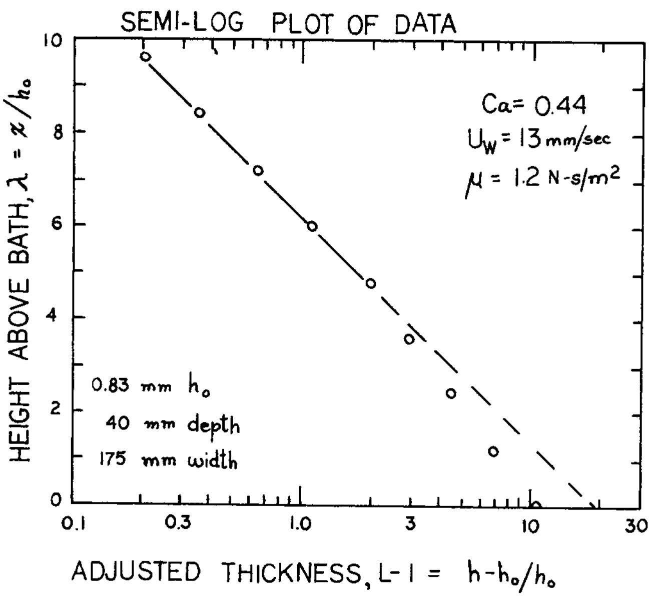

Fig. 2. Deviation of the lower meniscus: Semi-log plot of an $L(\lambda)$ profile, run 312B. The upper profile is $L = 1 + 19.5 \exp.(-0.48\lambda)$. The data value for L_B is 11.7. The deviation $\triangle$ of the extrapolated L_B value (of 19.5) is 8.8 or 42%.

STAGNATION POINT

Lee and Tallmadge (1972a) have predicted that the location of the deep-bath stagnation point varies with speed and Ca, as follows

$$\frac{h_{ST}}{h_0} = 3 - h_0^2 \left[\frac{\rho g}{\mu U_w}\right] = 3 - T_0^2 \quad (\zeta) \qquad (6)$$

The relation between thickness h_0 (or T_0) and Ca has been reported by Van Rossum (1958), Groenveld (1970), Soroka and Tallmadge (1971) and others.

COMPLETE PROFILES

A more general expression is needed to describe the lower region and the complete profile. One such expression for the data of Figure 2 is

$$L - 1 = 19.5 \exp(-0.48\lambda) - 9.0 \exp(-0.76\lambda) \qquad (7)$$

Equation (7), which was obtained by fitting the deviations in the lower meniscus, describes $(L - 1)$ or $(h - h_0)$ within 10% for the entire range of λ measured of 0 to 8. (Lee and Tallmadge, 1972c).

SUMMARY

Meniscus profiles have been measured for a range of coating speeds and bath depths. Description of these profiles has been discussed.

ACKNOWLEDGMENT

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Sufficient Conditions for Uniqueness and the Local Asymptotic Stability of Adiabatic Tubular Reactors

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The stability of adiabatic tubular reactor with axial mixing (ATRAM) has been studied first numerically by Raymond and Amundson (1964) and later by a number of investigators (Amundson, 1965; Nishimura and Matsubara, 1969; Berger and Lapidus, 1968; Clough and Ramirez, 1972) by solving a set of ordinary differential equations, an integral equation, or an inequality condition, all of which require the steady state profiles. Thus, their methods must also rely upon numerical solutions. The uniqueness of the steady state has been investigated by Luss and Amundson (1967) and Luss (1968) for an ATRAM and by Matsuyama (1970) for an ATRAM with recycle. The objective of this study is to obtain, using the approach used by Clough and Ramirez (1972), more convenient stability and uniqueness conditions than those currently available, that is, in terms of inlet or outlet temperature and/or system parameters. We also consider the case of equal Peclet number as a special case of the more general case of unequal Peclet numbers.

SUFFICIENT CONDITION FOR STABILITY IN TERMS OF STEADY STATE PROFILES

Consider an adiabatic tubular reactor with axial mixing (ATRAM) in which a single first-order, irreversible reaction is taking place. The dimensionless heat and mass balance equations are (Clough and Ramirez, 1972)

$$\frac{\partial n}{\partial t} = \frac{p_M}{p_H}\frac{\partial^2 n}{\partial x^2} - p_M\frac{\partial n}{\partial x} + B_1 f \tag{1}$$

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} - p_M\frac{\partial y}{\partial x} - B_2 f \tag{2}$$

with boundary conditions

$$-\frac{\partial n(0)}{\partial x} = p_H\,[1 - n(0)] \quad -\frac{\partial y(0)}{\partial x} = p_M\,[1 - y(0)] \tag{3}$$

$$\frac{\partial n(1)}{\partial x} = 0 \quad \frac{\partial y(1)}{\partial x} = 0 \tag{4}$$

The sufficient condition for stability as derived by Clough and Ramirez (1972) and corrected by Liou et al. (1972) is

$$Q(x) = B_1 f_n(x)/p_H p_M < 1/4 \quad \text{for all } x \in [0, 1] \tag{5}$$

where

$$f_n(x) = \frac{\partial f}{\partial n}\bigg|_{s.s.} = \frac{q y_s(x)}{n_s^2(x)}\, e^{-q/n_s(x)} \tag{6}$$

It is noted that $f_n(x)$ is determined from the steady state solutions of Equations (1) through (4).

Clough and Ramirez (1972) and Nishimura and Matsubara (1969) carried their stability criteria to this point, that is, in terms of steady state profiles. Starting from this point we further develop criteria in terms of exit or inlet temperature and also in terms of system parameters only. We do this by estimating the supremum of $Q(x)$ from the steady state equations.

Considering Inequality (5) we have

$$\underset{0 \leq x \leq 1}{\operatorname{Sup}}\; f_n(x) \leq \frac{p_H p_M}{4\,B_1} \tag{7}$$

Therefore, the problem is reduced to the estimation of $\operatorname{Sup} f_n(x)$.

SUFFICIENT CONDITION FOR STABILITY IN TERMS OF THE EXIT OR INLET TEMPERATURE

Two different cases are considered, Cases I and II.

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